

CVEN3502 Hydraulics

A study summary covering Open Channel Flow, Pumps, and Pipe Networks

This summary follows the teaching sequence: uniform flow & specific energy → channel transitions → hydraulic jumps → gradually varied flow → pump characteristics & cavitation → pump placement → pipe networks (Hardy-Cross). The single thread running through the open-channel half: identify which branch/regime you're on (sub- or supercritical, mild or steep slope), then apply energy or momentum conservation as appropriate.

1. Specific Energy & Critical Flow

The core idea

In open channel flow the water surface is unknown — depth y becomes the main variable. Specific energy is total head measured relative to the channel bed rather than a fixed datum:

$$E = y + v^2/(2g) \quad \text{compare to:} \quad H = z + y + v^2/(2g) = z + E$$

If total head H is constant (no friction loss) and the bed elevation z rises, E must fall to compensate, and vice versa. This one relationship ($z \uparrow \Rightarrow E \downarrow$) underlies almost everything in channel transitions.

Alternate depths & the E–y curve

For a rectangular channel with unit-width discharge $q = Q/B$:

$$E = y + q^2/(2gy^2) \quad \text{— cubic in } y \text{ for fixed } E, q$$

This cubic has two positive real roots for a given E : the **alternate depths** — one deep/slow, one shallow/fast. Velocity head dominates at small y ; depth dominates at large y ; between them E has a minimum.

Critical depth & the Froude number

$$y_c = (q^2/g)^{1/3} \quad E_c = (3/2) y_c \quad Fr = v / \sqrt{gy}$$

- **$Fr = 1$** ($y = y_c$): critical flow — minimum specific energy for given q
- **$Fr < 1$** ($y > y_c$): **subcritical** — deep, slow, tranquil (upper branch of E–y curve)
- **$Fr > 1$** ($y < y_c$): **supercritical** — shallow, fast (lower branch)

2. Channel Transitions

A smooth bed hump or channel contraction forces flow to move along the E–y curve. Because subcritical and supercritical flow sit on opposite branches, the same physical change produces *opposite* surface responses:

Approach flow	Bed rises (hump)	Bed narrows (contraction)
Subcritical ($y > y_c$)	Water surface DIPS	Depth drops toward y_c
Supercritical ($y < y_c$)	Water surface RISES	Depth rises toward y_c

Choking

Push a hump or contraction far enough and the flow is forced exactly through y_c at the critical section — push further and no real solution exists. The flow **chokes**: for subcritical approach flow, this forces the **upstream depth to rise** (a backwater effect), storing extra energy so there is enough head to force the flow through the throat. This is a useful design feature — critical-flow sections (venturi flumes, broad-crested weirs) give a known, fixed depth–discharge relationship, which is why they're used to measure flow.

3. Hydraulic Jumps

A hydraulic jump is the *violent* route from supercritical to subcritical flow (as opposed to the smooth transitions above). It is turbulent, 3D, and dissipates up to 85% of the approach flow's energy — so **energy is NOT conserved** and the specific energy diagram cannot be used. Instead, use **momentum + continuity**, which hold even through turbulent, lossy processes.

$$\text{Sequent depth ratio: } y_2/y_1 = \frac{1}{2}(\sqrt{1 + 8Fr_1^2} - 1)$$

$$\text{Energy lost: } \Delta E = (y_2 - y_1)^3 / (4y_1 y_2)$$

Continuity gives $q_1 = q_2 = v_1 y_1 = v_2 y_2$, which lets you convert between v , y and Fr on either side once you know the sequent depth ratio.

- Jumps are classified by Fr_1 — from undular ($Fr_1 = 1-1.7$) to steady/strong ($Fr_1 = 4.5-9$)
- Deliberately used in **stilling basins** below spillways/sluice gates to dissipate kinetic energy and prevent scour

4. Gradually Varied Flow (GVF)

GVF describes how depth changes slowly over long distances, as flow approaches or departs from normal depth y_o . The governing equation:

$$dy/dx = (S_o - S_f) / (1 - Fr^2)$$

S_o = bed slope (constant); S_f = friction slope (what slope would sustain the current v , y as uniform flow — equals S_o exactly when $y = y_o$).

Classifying a profile: two comparisons

- **y vs y_o** → sign of numerator ($S_o - S_f$)
- **y vs y_c** → sign of denominator ($1 - Fr^2$)

Zone	vs y_o , y_c	dy/dx sign	Behaviour
Type 1	y above both	+ / + = +	Depth increases downstream (backwater), e.g. ponding above a weir
Type 2	y between	mixed = −	Depth decreases downstream (drawdown)
Type 3	y below both	− / − = +	Depth increases downstream, supercritical, accelerating toward a jump

Slope classification: **Mild** ($y_o > y_c$, normal flow subcritical), **Steep** ($y_o < y_c$, normal flow supercritical), **Critical** ($y_o = y_c$), **Horizontal & Adverse** (y_o undefined). 12 standard profiles combine slope type (M, S, C, H, A) with zone type (1, 2, 3).

Key rules

- Profiles never cross y_o or y_c — approached asymptotically
- Subcritical flow is controlled from downstream → calculate **right to left**

- Supercritical flow is controlled from upstream → calculate **left to right**
- As $Fr \rightarrow 1$, $dy/dx \rightarrow \infty$ (near-vertical approach to critical depth — signature of a jump or free-fall)

Computed using the **Standard Step Method** (find y at a given x , iterative) or the **Direct Step Method** (find x at a given y , prismatic channels only): $\Delta x = (y_{i+1}^3 - y_i^3) / (S_o - S_{f,i})$.

5. Pumps

A pump adds energy to a system — it raises total head rather than losing it to friction. Rotodynamic pumps (centrifugal/radial, axial, mixed flow) are characterized experimentally by curves plotted against discharge Q :

- H_p - Q : head added — slopes downward (more flow, less head available)
- η - Q : efficiency — peaks at the **Best Efficiency Point (BEP)**; operate near BEP
- P_{in} - Q : input power required
- $NPSH_R$ - Q : required net positive suction head (cavitation limit)

Operating point

The **system curve** (static lift + friction/local losses, which grow with Q) slopes upward. It intersects the pump curve at the **operating point** — the only Q , H combination satisfying both the pump and the pipeline simultaneously. Adding losses (fittings, longer pipe) steepens the system curve → operating point moves to **lower Q , higher H** .

Cavitation & NPSH

Fluid accelerates sharply at the impeller inlet, dropping pressure. If pressure falls to the vapour pressure p_v , the water boils (pressure-boiling, not heat), forming bubbles that collapse violently downstream — **cavitation**, responsible for ~90–99% of pump failures.

$$NPSH_A = p_{atm}/(\rho g) - H_L - z_s - p_v/(\rho g) \quad \text{Avoid cavitation: } NPSH_A \geq NPSH_R$$

Static case (no flow), 20°C, sea level: maximum theoretical suction lift $z_s \approx 10.1$ m — a hard physical ceiling set by atmospheric pressure, independent of pump power.

Cavitation risk increases with:

- **Raising the pump** above the water source ($z_s \uparrow$)
- **Longer, narrower, rougher** suction pipe or extra fittings ($H_L \uparrow$)
- **Increasing Q** — hurts on both fronts at once (higher v_s and higher H_L)

Practical design response: keep the pump low and close to the source, use a short, wide, smooth suction pipe.

Pumps in series vs. parallel

	Series	Parallel
Flow	$Q = Q_A = Q_B$	$Q_t = Q_A + Q_B$
Head	$H_t = H_A + H_B$	$H_t = H_A = H_B$
Use case	High head, low flow	High flow, low head; redundancy

In parallel, all pumps share the same H , so **mismatched pumps are a real problem** — use identical pumps. Graphical solution: build a combined curve (add heads for series, add flows for parallel), intersect with the system curve to get the overall operating point, then go back to a single pump's curve to read its individual Q , H , power, and $NPSH_R$.

6. Pipe Networks — Hardy-Cross Method

A pipe network (multiple pipes, junctions, closed loops) must satisfy two conditions everywhere:

- **Continuity at every junction:** $\Sigma Q = 0$ (flow in = flow out)
- **Head loss around every closed loop sums to zero** — total head H at any point has only one value, regardless of path taken to reach it

Head loss is nonlinear in Q ($h_f = KQ|Q|$, from Darcy-Weisbach), so the system can't be solved directly — Hardy-Cross is an iterative guess-and-correct scheme:

- 1. **Guess** flows satisfying junction continuity (easy to arrange)
- 2. Loop head balance generally won't hold yet
- 3. Compute a flow correction for each loop: $\Delta Q = -\Sigma KQ|Q| / \Sigma 2K|Q|$
- 4. Apply ΔQ to every pipe in that loop (keeps continuity intact, nudges toward head balance)
- 5. Repeat across all loops — a pipe shared between two loops is corrected by both — until ΔQ is negligible

Sign convention: use $h_f = KQ|Q|$ (not KQ^2) so that direction is preserved — clockwise flows positive, counter-clockwise negative. If clockwise head losses exceed counter-clockwise ones, ΔQ comes out negative: this reduces clockwise flows and increases counter-clockwise flow magnitude, correctly rebalancing the loop.

Practical extras

- **Local losses** are folded in as an equivalent length of pipe
- A single reservoir feeding the network = constant-head boundary condition
- Two+ reservoirs at different levels need a **pseudo-loop** connecting them, since their head difference drives flow
- Total head H is always unique at a junction; pressure head $P/(\rho g)$ is **not** necessarily unique if pipes of different diameter meet there (different velocity heads)

Summary map: Open channel — uniform flow → specific energy → transitions → hydraulic jumps → gradually varied flow. Closed conduit — pump characteristics → operating point → cavitation/NPSH → series/parallel placement → pipe networks (Hardy-Cross). The recurring skill: identify the regime (sub/supercritical via Fr ; mild/steep via y_o vs y_c ; clockwise/counter-clockwise in a loop), then apply energy or momentum conservation accordingly.